THE HEATING OF BUTT-JOINED ORTHOTROPIC PLATES BY HEAT SOURCES WITH HEAT EXCHANGE

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UDC 539.377

A general solution is given for a nonstationary problem in heat conductivity and the corresponding quasi-static problem in thermoelasticity for compound semi-infinite plates heated by heat sources in the intermediate layer.

1. Conditions for the Thermomechanical Contact of Orthotropic Plates. Let two orthotropic plates of different materials be joined by a thin orthotropic layer between them (Fig. 1). Heat sources are distributed throughout the volume of the system, heat exchange with the external medium takes place across its surface in accordance with Newton's law, and there is ideal thermal contact between the plates and the intermediate layer [1].

In this case, to determine the nonstationary temperature field in the plates we have the equation of thermal conductivity [2] and the initial condition

$$L_iT_i-(\beta_iT_i+\beta_i^*T_i^*)=-(w_i+\beta_it_c+\beta_i^*t_c^*),$$

$$L_iT_i^*-3\left[\left(\beta_i+\frac{4}{r_{zi}^*}\right)T_i^*+\beta_i^*T\right]=-3(w_i^*+\beta_it_c^*+\beta_i^*t_c), \tag{1}$$

$$T_i=T_i^*=0 \quad \text{for} \quad \tau=0, \quad i=1,\ 2, \tag{2}$$

the conditions for the nonideal thermal contact of compound orthotropic plates, which we obtain, as for isotropic plates without heat sources [3], in the form

$$(p_s^2-\beta_s^0)(T_1+T_2)+2\left(\lambda_{x1}^0\frac{\partial T_1}{\partial x}-\lambda_{x2}^0\frac{\partial T_2}{\partial x}\right)-\beta_s^{*0}(T_1^*+T_2^*)=-2q_s, \tag{3}$$

$$(p_s^2-3B_s^0)(T_1^*+T_2^*)+2\left(\lambda_{x1}^0\frac{\partial T_1^*}{\partial x}-\lambda_{x2}^0\frac{\partial T_2^*}{\partial x}\right)-3\beta_s^{*0}(T_1+T_2)=-6g_s \quad (x=0),$$

$$\left(p_s^2-\frac{12}{r_{xs}^{*0}}-\beta_s^0\right)(T_1-T_2)+6\left(\lambda_{x1}^0\frac{\partial T_1}{\partial x}+\lambda_{x2}^0\frac{\partial T_2}{\partial x}\right)-\beta_s^{*0}(T_1^*-T_2^*)=-2q_s^*,$$

$$\left(p_s^2-\frac{12}{r_{xs}^{*0}}-3B_s^0\right)(T_1^*-T_2^*)+6\left(\lambda_{x1}^0\frac{\partial T_1^*}{\partial x}+\lambda_{x2}^0\frac{\partial T_2^*}{\partial x}\right)-3\beta_s^{*0}(T_1-T_2)=-6g_s^*$$

and the conditions at the end faces M_j [2, 3], where

$$L_i=\lambda_{xi}^0\frac{\partial^2}{\partial x^2}+\lambda_{yi}^0\frac{\partial^2}{\partial y^2}-C_i^0\frac{\partial}{\partial\tau},\quad p_s^2=\lambda_{ys}^0\frac{\partial^2}{\partial y^2}-C_s^0\frac{\partial}{\partial\tau},$$

$$T_i=\frac{1}{2\delta}\int_{-\delta}^{\delta}t_idz,\quad T_i^*=\frac{3}{2\delta^2}\int_{-\delta}^{\delta}zt_idz,\quad r_{zi}^*=\frac{2\delta}{\lambda_{zi}},$$

$$\lambda_{xi}^0=2\lambda_{xi}\delta,\quad \lambda_{yi}^0=2\lambda_{yi}\delta,\quad C_i^0=2C_i\delta,\quad \beta_i=\frac{1}{r_{zi}^+}+\frac{1}{r_{zi}^-},$$

$$\beta_i^*=\frac{1}{r_{zi}^+}-\frac{1}{r_{zi}^-},\quad i=1,\ 2;\quad t_c=\frac{t_c^++t_c^-}{2},\quad t_c^*=\frac{t_c^+-t_c^-}{2},\quad C_s^0=C_sF,$$

L'vov Institute of Physics and Mechanics, Academy of Sciences of the USSR. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol.17, No. 6, pp. 1067-1075, December, 1969. Original article submitted November 12, 1968.

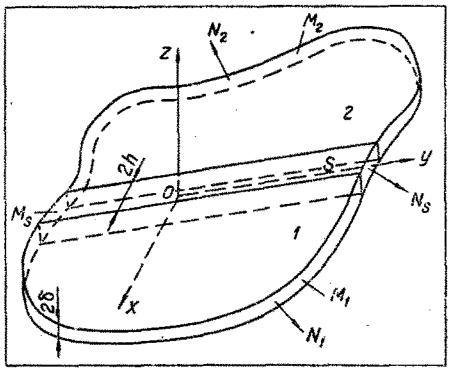

Fig. 1. Plates (1, 2) butt-joined by a thin layer.

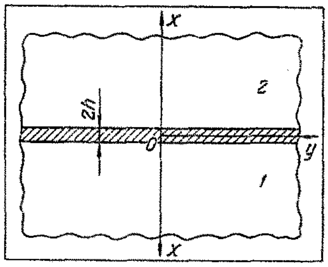

Fig. 2. Compound semi-infinite plates (1, 2).

$$\lambda_{ys}^0 = \lambda_{ys} F, \quad B_s^0 = \beta_s^0 + \frac{4}{r_{zs}^{*0}}, \quad r_{zs}^{*0} = \frac{\delta}{\lambda_z h}, \quad r_{xs}^{*0} = \frac{h}{\lambda_x \delta},$$

$$q_s = \int_{-h}^{h} Q_s dx, \quad q_s^* = \frac{3}{h}\int_{-h}^{h} xQ_s dx, \quad g_s = \int_{-h}^{h} Q_s^* dx,$$

$$g_s^* = \frac{3}{h}\int_{-h}^{h} xQ_s^* dx, \quad Q_s = w_s + \beta_s t_c + \beta_s^* t_c^*,$$

$$Q_s^* = w_s^* + \beta_s t_c^* + \beta_s^* t_c, \quad \beta_s^0 = 2\beta_s h, \quad \beta_s^{*0} = 2\beta_s^* h, \quad F = 4h\delta,$$

$$w_j = \int_{-\delta}^{\delta} W_j dz, \quad w_j^* = \frac{1}{\delta}\int_{-\delta}^{\delta} zW_j dz, \quad j = 1,\ 2,\ s.$$

When the thermal conductivity problem is symmetric about the plane z = 0 [4], instead of (1)-(3), we obtain respectively

$$L_i T_i - \frac{2}{r_{zi}}(T_i - t_c) = -w_i, \tag{4}$$

$$T_i = 0 \quad \text{for} \quad \tau = 0; \tag{5}$$

$$\left(p_s^2 - \frac{2}{r_{zs}^0}\right)(T_1 + T_2) + 2\left(\lambda_{x1}^0 \frac{\partial T_1}{\partial x} - \lambda_{x2}^0 \frac{\partial T_2}{\partial x}\right) = -2q_s, \tag{6}$$

$$\left(p_s^2 - \frac{12}{r_{xs}^{*0}} - \frac{2}{r_{zs}^0}\right)(T_1 - T_2) + 6\left(\lambda_{x1}^0 \frac{\partial T_1}{\partial x} + \lambda_{x2}^0 \frac{\partial T_2}{\partial x}\right) = -2q_s^* \quad (x = 0),$$

where

$$r_{zs}^0 = \frac{r_{zs}}{2h}, \quad q_s = \int_{-h}^{h}\left(w_s + \frac{2t_c}{r_{zs}}\right)dx, \quad q_s^* = \frac{3}{h}\int_{-h}^{h} x\left(w_s + \frac{2t_c}{r_{zs}}\right)dx.$$

If in (6) we ignore the products $r_{xs}^{*0}\lambda_{ys}^0$, $r_{xs}^{*0}C_s^0$, $r_{xs}^{*0}\alpha_{zs}^0$, we obtain the conditions

$$\lambda_{x1}^0 \frac{\partial T_1}{\partial x} - \lambda_{x2}^0 \frac{\partial T_2}{\partial x} = -w_s^0,$$

$$2(T_1 - T_2) - r_{xs}^{*0}\left(\lambda_{x1}^0 \frac{\partial T_1}{\partial x} + \lambda_{x2}^0 \frac{\partial T_2}{\partial x}\right) = r_{xs}^{*0} w_s^{*0} \quad (x = 0),$$

where

$$w_s^0 = \int_{-h}^{h} w_s dx, \quad w_s^{*0} = \frac{1}{h}\int_{-h}^{h} xw_s dx, \quad r_{zs}^0 = \frac{1}{\alpha_{zs}^0}, \quad \alpha_{zs}^0 = 2h\alpha_{zs}.$$

Corresponding to the conditions (6) for nonideal thermal contact, we obtain the mechanical conditions in the same way as were obtained [2] the boundary conditions at the edge of orthotropic plates supported by a thin orthotropic column:

$$\frac{\partial}{\partial y}\left(\frac{\nu_{x1}}{E_{x1}}\sigma_{x1}-\frac{\sigma_{y1}}{E_{y1}}-\alpha^t_{y1}T_1+\alpha^t_{ys}\theta_s\right)=\frac{2\delta}{g_{ys}}(\sigma_{xy1}-\sigma_{xy2}), \tag{7}$$

$$\frac{\partial^2}{\partial y^2}\left[\left(\frac{1}{G_1}-\frac{\nu_{x1}}{E_{x1}}\right)\frac{\partial\sigma_{xy1}}{\partial y}-\frac{\partial}{\partial x}\left(\frac{\sigma_{y1}}{E_{y1}}+\alpha^t_{y1}T_1\right)+\alpha^t_{ys}\theta^*_s\right]=\frac{2\delta}{g^*_{ys}}(\sigma_{x1}-\sigma_{x2}),\quad u_1=u_2,\quad v_1=v_2\quad (x=0),$$

where

$$g_{ys}=E_{ys}F,\quad g^*_{ys}=E_{ys}I,\quad I=\frac{4}{3}\delta h^3,$$

$$\theta_s=\frac{1}{2h}\int_{-h}^{h}T_s dx=\frac{T_1+T_2}{2},\quad \theta^*=\frac{3}{2h^3}\int_{-h}^{h}xT_3dx=\frac{T_1-T_2}{2h}.$$

Conditions (6) and (7) are called the conditions for the nonideal thermomechanical contact of orthotropic plates joined by a thin orthotropic layer.

If we put λ^0_{ys}, C^0_s, r^0_{zs}, $\alpha^0_{zs}=1/r^0_{zs}$, w^0_s, w^{*0}_s, g_{ys}, g^*_{ys} equal to zero in (6), (7), we obtain the conditions for the ideal thermomechanical contact of orthotropic plates:

$$\begin{gathered}\lambda_{x1}\frac{\partial T_1}{\partial x}=\lambda_{x2}\frac{\partial T_2}{\partial x},\quad T_1=T_2,\\ \sigma_{x1}=\sigma_{x2},\quad \sigma_{xy1}=\sigma_{xy2},\quad u_1=u_2,\quad v_1=v_2\quad \text{for}\quad x=0.\end{gathered} \tag{8}$$

We note that if $w^0_s \neq 0$, the first condition of (6) has the form

$$\lambda_{x1}\frac{\partial T_1}{\partial x}=\lambda_{x2}\frac{\partial T_2}{\partial x}-\varepsilon W^0_s\quad \text{for}\quad x=0. \tag{9}$$

Condition (9) can be used to compute the thermal processes when plates of different materials are butt welded.

2. Temperature Stresses in Compound Semi-Infinite Plates. Consider orthotropic plates joined by a thin orthotropic layer between them with heat sources only in the intermediate layer (Fig. 2). In Eq. (4), where $t_c = 0$, $w_i = 0$, and conditions (6), with initial condition (5), we make an integral Fourier transform with respect to y and a Laplace transform with respect to τ, solve the resulting equations with the transformed boundary conditions, and find the following expression for the Fourier–Laplace transforms of the temperature system:

$$\overline{T}_i=A_i\exp(-\gamma_i x), \tag{10}$$

where

$$A_1=2\left[(\gamma_0^2+6\lambda^0_{x2}\gamma_2)\overline{w}^0_s+3(\gamma_s^2+2\lambda^0_{x2}\gamma_2)\overline{w}^{*0}_s\right]D^{-1},$$

$$A_2=2\left[(\gamma_0^2+6\lambda^0_{x1}\gamma_1)\overline{w}^0_s-3(\gamma_s^2+2\lambda^0_{x1}\gamma_1)\overline{w}^{*0}_s\right]D^{-1},$$

$$\gamma_i^2=\frac{p}{a_i}+k_i^2\eta^2+\varkappa_i^2,\quad \gamma_0^2=\gamma_s^2+\frac{12}{r^{*0}_{xs}},\quad a_i=\frac{\lambda_{xi}}{C_i},$$

$$k_i^2=\frac{\lambda_{yi}}{\lambda_{xi}},\quad \varkappa_i^2=\frac{\alpha_{zi}}{\lambda_{xi}\delta},\quad \alpha_{zi}=\frac{1}{r_{zi}},\quad \gamma_s^2=\lambda^0_{ys}\eta^2+C^0_s p+\frac{2}{r^0_{zs}},$$

$$\overline{F}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dy\int_0^{\infty}F\exp(i\eta y-p\tau)\,d\tau,$$

$$D=(6\lambda^0_{x2}\gamma_2+\gamma_0^2)(2\lambda^0_{x1}\gamma_1+\gamma_s^2)+(6\lambda^0_{x1}\gamma_1+\gamma_0^2)(2\lambda^0_{x2}\gamma_2+\gamma_s^2).$$

If we return from (10) by the inversion theorem for Fourier–Laplace transformations to the originals, we obtain the general solution for the nonstationary thermal conductivity problem for the system under consideration. The stressed–deformed state due to the temperature field is defined by the familiar equation [5]

$$\sigma_{xi} = \frac{\partial^2 \Phi_i}{\partial y^2}, \quad \sigma_{yi} = \frac{\partial^2 \Phi_i}{\partial x^2}, \quad \sigma_{xyi} = -\frac{\partial^2 \Phi_i}{\partial x \partial y}; \tag{11}$$

$$e_{xi} = \frac{\sigma_{xi}}{E_{xi}} - \frac{\nu_{xi}}{E_{xi}} \sigma_{yi} + \alpha^t_{xi} T_i,$$
$$e_{yi} = -\frac{\nu_{xi}}{E_{xi}} \sigma_{xi} + \frac{\sigma_{yi}}{E_{yi}} + \alpha^t_{yi} T_i, \quad e_{yxi} = \frac{1}{G_i} \sigma_{yxi} \tag{12}$$

subject to (7). Here the Φ_i is the stress function satisfying the equation

$$\frac{\partial^4 \Phi_i}{\partial x^4} + 2s_{0i} \frac{\partial^4 \Phi_i}{\partial x^2 \partial y^2} + s_i \frac{\partial^4 \Phi_i}{\partial y^4} = -E_{yi} \left(\alpha^t_{yi} \frac{\partial^2 T_i}{\partial x^2} + \alpha_{xi} \frac{\partial^2 T_i}{\partial y^2} \right), \tag{13}$$

$$s_{0i} = -\nu_{yi} + \frac{E_{yi}}{2G_i}, \quad s_i = \frac{E_{yi}}{E_{xi}}, \quad e_{xi} = \frac{\partial u_i}{\partial x}, \quad e_{yi} = \frac{\partial v_i}{\partial y},$$
$$e_{yxi} = \frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}.$$

If we make a Fourier–Laplace transformation of (11)-(13), (7), as in [2, 6], we obtain expressions for the transforms of the temperature stresses when the characteristic equation $\mu_i^4 - 2s_{0i}\mu_i^2 + s_i = 0$ has real and unequal roots:

$$\bar{\sigma}_{xi} = -\eta^2 [C_i \exp(-\eta_{Ii} x) + D_i \exp(-\eta_{IIi} x) + \Omega_i \bar{T}_i],$$
$$\bar{\sigma}_{yi} = C_i \eta^2_{Ii} \exp(-\eta_{Ii} x) + D_i \eta^2_{IIi} \exp(-\eta_{IIi} x) + \Omega_i \gamma_i^2 \bar{T}_i, \tag{14}$$
$$\bar{\sigma}_{yxi} = -i\eta [C_i \eta_{Ii} \exp(-\eta_{Ii} x) + D_i \eta_{IIi} \exp(-\eta_{IIi} x) + \Omega_i \gamma_i \bar{T}],$$

where

$$\Omega_i = \frac{\alpha^t_{xi}\eta^2 - \alpha^t_{yi}\gamma_i^2}{\eta^4 s_i - 2s_{oi}\eta^2\gamma_i^2 + \gamma_i^4} E_{yi}, \quad \eta_{Ii} = \mu_{Ii}|\eta|, \quad \eta_{IIi} = \mu_{IIi}|\eta|,$$

$$C_i = \frac{c_i}{d_0}, \quad D_i = \frac{d_i}{d_0}, \quad \bar{\sigma}_{xy} = \bar{\sigma}_{yx};$$

$$d_0 = B(\eta_{I1}, \eta_{II1}) B(\eta_{I2}, \eta_{II2}) + \xi(\eta_{I1}, \eta_{I2}) \xi(\eta_{II2}, \eta_{II1}) - \xi(\eta_{I1}, \eta_{II2}) \xi(\eta_{I2}, \eta_{II1}) - \xi(\eta_{II1}, \eta_{I2}) \xi(\eta_{II2}, \eta_{I1})$$
$$+ \xi(\eta_{II1}, \eta_{II2}) \xi(\eta_{I2}, \eta_{I1}) + S(\eta_{II2}, \eta_{I2}) S(\eta_{II1}, \eta_{I1});$$

$$c_1 = P(\eta_{II1}) B(\eta_{I2}, \eta_{II2}) + l(\eta_{I2}) \xi(\eta_{II2}, \eta_{II1}) - l(\eta_{II2}) \xi(\eta_{I2}, \eta_{II1})$$
$$- \zeta(\eta_{I2}) \xi(\eta_{II1}, \eta_{II2}) + \zeta(\eta_{II2}) \xi(\eta_{II1}, \eta_{I2}) + R(\eta_{I1}) S(\eta_{II2}, \eta_{I2});$$

$$d_1 = P(\eta_{I1}) B(\eta_{I2}, \eta_{II2}) + l(\eta_{I2}) \xi(\eta_{II2}, \eta_{I1}) - l(\eta_{II2}) \xi(\eta_{I2}, \eta_{I1})$$
$$- \zeta(\eta_{I2}) \xi(\eta_{I1}, \eta_{II2}) + \zeta(\eta_{II2}) \xi(\eta_{I1}, \eta_{I2}) + R(\eta_{I1}) S(\eta_{II2}, \eta_{I2});$$

$$c_2 = P(\eta_{II1}) \xi(\eta_{II2}, \eta_{I1}) - P(\eta_{I1}) \xi(\eta_{II2}, \eta_{I1}) + l(\eta_{II2}) S(\eta_{II1}, \eta_{I1})$$
$$+ R(\eta_{I1}) \xi(\eta_{II1}, \eta_{II2}) - R(\eta_{II1}) \xi(\eta_{I1}, \eta_{II2}) + \zeta(\eta_{II2}) B(\eta_{I1}, \eta_{II1});$$

$$d_2 = P(\eta_{II1}) \xi(\eta_{I2}, \eta_{I1}) - P(\eta_{I1}) \xi(\eta_{I2}, \eta_{II1}) + l(\eta_{I2}) S(\eta_{II1}, \eta_{I1})$$
$$+ R(\eta_{I1}) \xi(\eta_{II1}, \eta_{I2}) - R(\eta_{II1}) \xi(\eta_{I1}, \eta_{I2}) + \zeta(\eta_{I2}) B(\eta_{I1}, \eta_{II1});$$

$$B(\eta_{ki}, \eta_{mi}) = f(\eta_{mi}) \varphi(\eta_{ki}) - f(\eta_{ki}) \varphi(\eta_{mi});$$

$$P(\eta_{k1}) = (V_1 - \chi_1 - \chi_2) \varphi(\eta_{k1}) - (\varphi_1 + \psi_1 - \psi_2) f(\eta_{k1});$$

$$\xi(\eta_{ki}, \eta_{mn}) = \frac{\eta_{mn}}{G_s} \varphi(\eta_{ki}) - \frac{f(\eta_{ki})}{G_s^*}, \quad G_s = \frac{g_{ys}}{2\delta}, \quad G_s^* = \frac{g^*_{ys}}{2\delta};$$

$$S(\eta_{IIi}, \eta_{In}) = \frac{\eta_{IIi} - \eta_{In}}{G_s G_s^*}, \quad f(\eta_{ki}) = \frac{\nu_{xi}}{E_{xi}} \eta^2 + \frac{\eta^2_{ki}}{E_{yi}} + \frac{\eta_{ki}}{G_s};$$

$$\varphi(\eta_{ki}) = \left[\frac{\eta^2_{ki}}{E_{yi}} + \left(\frac{\nu_{xi}}{E_{xi}} - \frac{1}{G_i} \right) \eta^2 \right] \eta_{ki} - \frac{1}{G_s^*}, \quad l(\eta_{k2}) = \frac{V_1 - \chi_1 - \chi_2}{G_s^*} + \frac{\eta_{k2}}{G_s}(-\varphi_1 - \psi_1 + \psi_2);$$

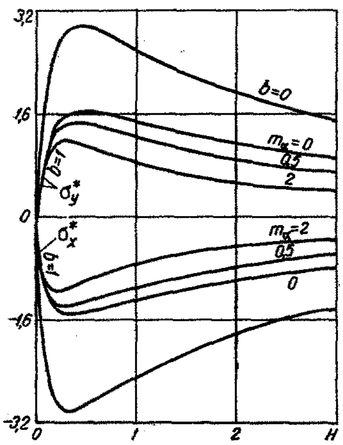

Fig. 3

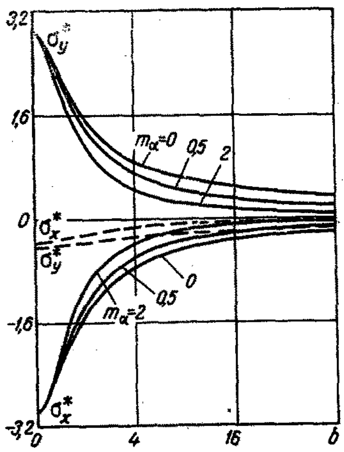

Fig. 4

Fig. 3. Temperature stresses as functions of H for b = 0 and 1, $m_\alpha = 0,\ 0.5$, and 2.

Fig. 4. Temperature stresses as functions of b for H = 0.34, $m_\alpha = 0,\ 0.5$, and 2.

$$R(\eta_{k1}) = \frac{V_2 - \chi_1 - \chi_2}{G_s^*} - \frac{\eta_{k1}}{G_s}(\varphi_2 - \psi_1 + \psi_2);$$

$$\zeta(\eta_{k2}) = (V_2 - \chi_1 - \chi_2)\,\varphi(\eta_{k2}) + (\varphi_2 - \psi_1 + \psi_2)\,f(\eta_{k2});$$

$$\varphi_i = \left\{\left[\frac{\gamma_i^2}{E_{yi}} + \eta^2\left(\frac{\nu_{xi}}{E_{xi}} - \frac{1}{G_i}\right)\right]\Omega_i + \alpha_{yi}^t\right\}\gamma_i\bar{t}_i, \quad \bar{t}_i = \bar{T}_i\,|_{x=0};$$

$$\psi_i = \left(\frac{\alpha_{ys}^t}{2h} - \frac{\Omega_i}{G_s^*}\right)\bar{t}_i, \quad \chi_i = \frac{\alpha_{ys}^t}{2} - \frac{\gamma_i}{G_s}\Omega_i, \quad V_i = \frac{\varphi_i}{\gamma_i} - \frac{\eta^2}{G_i}\Omega_i\bar{t}_i.$$

If we put $\mu_{\mathrm{I}i} = \mu_{\mathrm{II}i} = \mu_{0i}$ or $\mu_{\mathrm{I}i} = \mu_i + r_i i$, $\mu_{\mathrm{II}} = \mu_i - r_i i$ in (14), we find expressions for the transforms of the temperature stresses when the roots of the above equation are real and pairwise equal ($\pm\mu_{0i}$, $\mu_{0i} > 0$) or complex ($\mu_i \pm r_i i$, $-\mu_i \pm r_i i$, $\mu_i > 0$, $r_i > 0$).

If we return in (14) to the originals, we obtain the general solution of the quasistatic thermoelastic problem for the plate system under consideration.

We now assume that the density of the heat sources in the intermediate layer is

$$W_s = q\cos\omega y\,\delta(x,\ z),$$

the Fourier transformation of which has the form [7]

$$\bar{W}_s = q\sqrt{\frac{\pi}{2}}\,[\delta(\eta + \omega) + \delta(\eta - \omega)]\,\delta(x,\ z). \tag{15}$$

We substitute (15) in (14) and pass to the limit in these variables as $p \to 0$. If we then use the inversion theorem for the Fourier transformation to return in the expressions thus obtained to the originals, we obtain the temperature stresses in compound plates when the temperature conditions are stationary:

$$\sigma_{xi} = \bar{\sigma}_{xi}(\omega)\cos\omega y,$$
$$\sigma_{yi} = \bar{\sigma}_{yi}(\omega)\cos\omega y, \quad \sigma_{yxi} = -\bar{\sigma}_{yxi}(\omega)\,i\sin\omega y. \tag{16}$$

When both the plates are made of the same material, Eqs. (16) become

$$\sigma_x^* = -[C\exp(-\mu_{\mathrm{I}}X) + D\exp(-\mu_{\mathrm{II}}X) + \Omega_0 T_0]\cos Y,$$
$$\sigma_y^* = [C\mu_{\mathrm{I}}^2\exp(-\mu_{\mathrm{I}}X) + D\mu_{\mathrm{II}}^2\exp(-\mu_{\mathrm{II}}X) + \Omega_0\gamma^2 T_0]\cos Y, \tag{17}$$
$$\sigma_{xy}^* = -[C\mu_{\mathrm{I}}\exp(-\mu_{\mathrm{I}}X) + D\mu_{\mathrm{II}}\exp(-\mu_{\mathrm{II}}X) + \Omega_0\gamma T_0]\sin Y,$$
$$\sigma_x^* = \frac{4\sigma_x\lambda_x\delta\omega}{\alpha_y^t E_y q}, \quad C = \frac{c}{d_0}, \quad D = \frac{d}{d_0}, \quad Y = \omega y, \quad X = \omega x,$$

$$d = N\varphi(\mu_I) - Mf(\mu_I), \quad d_0 = f(\mu_I)\varphi(\mu_{II}) - f(\mu_{II})\varphi(\mu_I),$$

$$f(\mu_k) = \nu_y + \mu_k^2 + \frac{\mu_k}{Hm_E}, \quad \varphi(\mu_k) = \left(\nu_y + \mu_k^2 - \frac{E_y}{G}\right)\mu_k, \quad H = \omega h,$$

$$M = t_0[\Omega_0\varphi(\gamma) + \gamma], \quad N = t_0[\Omega_0 f(\gamma) + 1 - m_\alpha^t], \quad m_\alpha^t = \frac{\alpha_{ys}^t}{\alpha_y^t},$$

$$m_E = \frac{E_{ys}}{E_y}, \quad m_\lambda = \frac{\lambda_{ys}}{\lambda_x}, \quad b = \frac{\varkappa^2}{\omega^2}, \quad \gamma = \sqrt{k^2 + b}, \quad \varkappa^2 = \frac{\alpha_z}{\lambda_x\delta}, \quad m_\alpha = \frac{\alpha_{zs}}{\alpha_z},$$

$$T_0 = \exp(-X\gamma)[\gamma + H(m_\lambda + m_\alpha b)]^{-1}, \quad \Omega_0 = \frac{\dfrac{\alpha_x^t}{\alpha_y^t} - \gamma^2}{\gamma^4 - 2s_0\gamma^2 + s},$$

$$t_0 = T_0|_{x=0}, \quad \bar{\sigma}_{xi}(\omega) = \bar{\sigma}_{xi}(\eta)\Big|_{s=0,\ \eta=\omega,\ \bar{w}_s^{*0}=0,\ \bar{w}_s^0=q}, \quad c = Mf(\mu_{II}) - N\varphi(\mu_{II}).$$

Suppose both plates are made of glass Textolite KAST-V [8], while the intermediate layer is made of epoxide resin [9, 10]. If we substitute in (17) the values of the thermophysical and mechanical characteristics for the materials of the plate and the layer, for $x = 0$, $y = 0$ we obtain the following nondimensional temperature stresses:

$$\sigma_x^* = \frac{0.17}{d_0}\left[\left(\frac{1}{m_E H} + 1.83\right)M - 0.7069N\right] - \Omega_0 t_0,$$
$$\sigma_y^* = \frac{0.17}{d_0}\left[\left(\frac{0.83}{m_E H} + 0.12\cdot 1.83\right)M + 0.83\cdot 2.642N\right] + \Omega_0\gamma^2 t_0, \quad (18)$$

where

$$m_E^{-1} = 0.04449; \quad m_\alpha^t = 1.25, \quad m_\lambda = 0.9554, \quad \mu_I = 1, \quad \mu_{II} = 0.83,$$
$$\nu_y = 0.12; \quad \gamma = \sqrt{b + 1.6}, \quad \frac{E_y}{G} = 1.932, \quad \Omega_0 = \frac{b + 0.85}{(b + 0.6)(0.6889 - \gamma^2)},$$
$$d_0 = f(1)\varphi(0.83) - f(0.83)\varphi(1).$$

If in (17) we ignore the thermophysical and mechanical characteristics of the layer, i.e., we put m_λ, m_α, m_α^t, m_E equal to zero, for $x = 0$, $y = 0$, and plates of glass Textolite KAST-V we find that

$$\sigma_x^* = \Omega_0\left(1 - \frac{1}{\gamma} - \frac{\gamma^2}{1.5189}\right) - \frac{1}{1.5189},$$
$$\sigma_y^* = \gamma\Omega_0\left(1 - \frac{\gamma}{1.83}\right) - \frac{0.83\Omega_0 + 1}{1.83}. \quad (19)$$

From (18) we computed the temperature stresses σ_x^* and σ_y^* as functions of H, b, m_α as shown in Figs. 3, 4. For comparison, Fig. 4 also contains the temperature stresses (dotted lines) computed from (19). We see from the graphs that the intermediate layer has a significant effect on the temperature stresses in compound plates when there is heat exchange.

NOTATION

$t_i(x, y, z, \tau)$	is the temperature of the plates;
τ	is the time;
r_{zi}^*	is the internal thermal resistance of the plates in the z-direction;
$r_{zi}^{\pm} = 1/\alpha_{zi}^{\pm}$	is their resistance to heat exchange on the surfaces $z = \pm\delta$;
λ_{xi}, λ_{yi}	are the coefficients of thermal conductivity of the plates in the x and y directions;
C_i	is their heat capacities;
λ_{xi}^0, λ_{yi}^0, C_i^0	are the reduced coefficients of thermal conductivity in the x and y directions and the heat capacity;
$t_c^{\pm}$	is the temperature of the medium surrounding the surfaces $z = \pm\delta$ of the system;
λ_{ys}^0	is the reduced coefficient of thermal conductivity for the layer in the y direction;

C_s^0	is its reduced heat capacity;
F	is its cross-sectional area;
2δ	is the thickness of the system;
2h	is the thickness of the layer;
W_j	is the density of the heat sources in the plates and the layer;
r_{zs}^{*0}, r_{xs}^{*0}	is the internal contact thermal resistance of the layer in the z and x directions;
g_{ys}, g_{ys}^{*}	is the rigidity of the layer in tension (compression) and bending rigidity;
I	is the moment of inertia;
α_{ys}^{t}	is the temperature coefficient of linear expansion of the layer in the y direction;
α_{xi}^{t}, α_{yi}^{t}	are the temperature coefficients of linear expansion of the plates in the x and y directions;
σ_{xi}, σ_{yi}, σ_{yxi} and e_{xi}, e_{yi}, e_{yxi}	are the coefficients of the temperature stress tensor for the plates and of the deformation tensor;
u_i, v_i	are the displacements of the plates in the x and y directions;
E_{xi}, E_{yi}	are Young's moduli for tension (compression) of the plates in the x and y directions;
E_{ys}	is Young's modulus for tension (compression) for the layer;
ν_{xi}, ν_{yi}	are Poisson's coefficients for the plates, defining compression in the y direction and extension in x direction, and conversely;
G_i	is the shear modulus for the plates, defining the change in the angles between the x and y directions;
$\delta(x)$	is the Dirac delta function;

$\varepsilon = h/\delta$, $W_s^0 = 1/F\int_{-h}^{h} w_s dx$;

$\bar{\delta}_{xi}(\omega) = \bar{\delta}_{xi}(\eta)\Big|_{\eta=\omega;\ s=0;\ \bar{w}_s^{*0}=0;\ \bar{w}_s^{0}=q.}$

LITERATURE CITED

1. A. V. Lykov, The Theory of Thermal Conductivity [in Russian], Izd. Vysshaya Shkola, Moscow (1967).
2. Yu. M. Kolyano, Inzh. Zh., MTT, No. 3 (1968).
3. V. M. Gembara, Problems in the Mechanics of a Real Rigid Body [in Russian], No. 3, Izd. Naukova Dumka, Kiev (1964).
4. Ya. S. Podstrigach and Yu. M. Kolyano, Inzh.-Fiz. Zh., 7, No. 2 (1964).
5. V. Novatskii, Problems in Thermo-elasticity [in Russian], Izd. Akad. Nauk SSSR (1962).
6. Yu. M. Kolyano, Prikl. Mekhan., 3, No. 6 (1967).
7. I. M. Gel'fand and G. E. Shilov, Generalized Functions and Operations on Them [in Russian], Fizmatgiz (1959).
8. S. A. Ambartsumyan and S. M. Durgar'yan, Izv. Akad. Nauk SSSR, Mekhan. i Mashinostr., No. 3, 124 (1962).
9. K. S. Bolotina, Izv. Vuzov, Mashinostroenie, No. 12, 27 (1966).
10. V. V. Bolotin and K. S. Bolotina, Mekhanika Polimerov, No. 1, 138 (1967).